SUBLIMATION KINETICS OF A THIN COATING ON THE WALLS OF A SLOT SYSTEM

P. A. Novikov* and L. Ya. Lyubin

UDC 536.422:532.529.5

The linearized problem of phase transformations at the walls of a slot system with relatively small pressure differences is formulated. The solution is obtained in a quasisteady approximation.

Equation (11) of [1] describes the quasisteady phase transformations in a slot system when the intensity of the process is determined by the hydraulic drag of the system and the thermodynamic drag of the walls, and the external surfaces of the walls are maintained at constant temperature. If relatively small pressure differences are realized in the narrow gap between parallel plates of arbitrary shape in plan view ($\Pi_{max} - \Pi_{min} \leq \Pi_{min}$), the problem may be linearized by introducing a small perturbation $\Pi' = \Pi - \Pi_* = o(\Pi_*)$. After substituting the new variables $\xi = \omega_0^{-1/2}\xi_1$, $\eta = \omega_0^{-1/2}\eta_1$, $\Pi_1 = 1 + \Pi'/\Pi_*$ into Eq. (11) of [1], expanding the functions appearing there in Taylor series with respect to the small parameter Π'/Π_*, and retaining only the first-order terms in these expansions, an inhomogeneous Helmholtz equation is obtained

$$\nabla^2\Pi' + A_1\Pi' + A_0 = 0. \tag{1}$$

On the open (Γ_{2n+1}) and closed (Γ_{2n}) sections of the slotted contour, the following boundary conditions must hold

$$\Pi' \leqslant \Pi'_{2n+1} \text{ on } \Gamma_{2n+1}, \quad \frac{\partial\Pi'}{\partial N} = 0 \text{ on } \Gamma_{2n}. \tag{2}$$

This problem is uniquely solvable if the coefficient A_1 does not coincide with any of the eigenvalues of the corresponding homogeneous problem, which are positive numbers [2]. However, $A_1 < 0$, since $\varepsilon = O(Kn)$, and hence for the viscous and molecular-viscous conditions given in [1] and here, $\varepsilon \ll 1$.

An equation of the type in Eq. (1) obtained in [3] by the methods of molecular-kinetic theory describes the fields of different physical nature, say, the temperature field of the walls of a slot system over a wide range of Kn, if homogeneous sublimation (desublimation) occurs on account of heat supply (extraction) at the walls, and the transfer processes depend significantly on the phase resistance. Note that, in the latter case, A_1 is not always negative, which somewhat complicates the analysis of such problems.

This analogy permits the use of the analytical results of all the examples in [3] for the processes here considered. Attention will be confined to the two simplest problems: phase transformation at the walls of a narrow slot channel (length 2L) open on both sides and in the gap between two parallel disks (radius L). These problems are somewhat more complicated than those considered in [1], on account of the assumption that the phase transformations do not occur over the whole internal surface but are localized within the limits $|\xi| > \xi' \leq 1$, so that the dimensionless excess pressure Π' on the sections $\xi' < |\xi| < 1$ must satisfy the Laplace equation $\nabla^2\Pi' = 0$. Thus, the problem is reduced to solving an ordinary differential equation

$$\frac{d^2\Pi'}{d\xi^2} + \frac{\nu}{\xi}\frac{d\Pi'}{d\xi} - \xi_0^2\,\Pi' + A_0 = 0 \tag{3}$$

*Deceased.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 1, pp. 113-117, July 18, 1988. Original article submitted February 13, 1987.

 0022-0841/88/5501-0792 $12.50

with a piecewise-constant coefficient ξ_0^2 equal to A_1 when $|\xi| < \xi'$ and 0 when $\xi' < |\xi| < 1$.

The desired function Π' must satisfy a homogeneous boundary condition of the first kind in the section of the gap and also conditions of smoothness and continuity at the boundary of the phase-transformation zone. In addition, the obvious symmetry condition at $\xi = 0$ must be satisfied. Hence

$$\frac{d\Pi'(0)}{d\xi} = 0,\ \Pi'(\xi' - 0) = \Pi'(\xi' + 0),$$
$$\frac{d\Pi'(\xi' - 0)}{d\xi} = \frac{d\Pi'(\xi' + 0)}{d\xi};\ \Pi'(1) = 0. \tag{4}$$

In the phase-transformation zone $|\xi| < \xi'$ ($|\xi_*| < \xi_*'$)

$$\Pi' = A_*[1 - \Phi_\nu(\xi_*,\ \xi_*')];\ A_* = A_0/\xi_0^2 = (\Pi_*/\omega)\,\delta\Theta_w; \tag{5}$$

$$\Phi_0(\xi_*,\ \xi_*') = \frac{\operatorname{ch}\xi_*}{\operatorname{ch}\xi_*' + (\xi_0 - \xi_*')\operatorname{sh}\xi_*'}\quad (\xi_* = \xi_0\xi);$$
$$\Phi_1(\xi_*,\ \xi_*') = \frac{I_0(\xi_*)}{I_0(\xi_*') + \xi_*'(\ln\xi_0 - \ln\xi_*')\,I_1(\xi_*')}. \tag{6}$$

As in [3], the relations obtained may be used to determine the sublimation time of the coating applied in a thin homogeneous layer on the internal surface of the walls of a slot system if the process occurs quasisteadily. This is a natural assumption in considering the wear of coating material, which occurs very slowly, as a rule. In the general case, studying the kinetics of quasisteady sublimation of a thin layer at the walls of a slot device reduces to solving sufficiently complex nonlinear and nonsteady functional equations. However, as in [3], the singularity of the two examples considered is that, in these cases, the quasisteady sublimation process of a coating applied in a layer of thickness $\Delta \ll h$ occurs self-similarly. In fact, in view of Eq. (5) and also Eq. (7) from [1], with symmetric heat supply through both walls, the mass flux density of sublimate from the sections of the wall still having a coating ($\xi_* < \xi_*'$) is

$$J_\nu(\xi_*,\ \xi_*') = J_m\Phi_\nu(\xi_*,\ \xi_*'). \tag{7}$$

Together with Eq. (6), Eq. (7) means that here, as in the examples in [3], the flux density is always distributed according to the same law along the surface where the coating is still present, the area of which decreases over time, regardless of the position of the boundary $\xi_*'(t)$, although the profile $J_\nu[\xi_*,\ \xi_*'(t)]$ is deformed similarly over time. Hence, as in [3], the dimensionless time ϑ_ν of motion of the boundary of the phase-transformation zone from its initial position ξ_*^0 to 0 in the two examples considered is determined by the expressions

$$\vartheta_0(\xi_*^0) = \frac{1}{2}(\xi_*^0)^2 + (\xi_0 - \xi_*^0)(\xi_*^0 - \operatorname{th}\xi_*^0);$$
$$\vartheta_1(\xi_*^0) = \frac{1}{4}(\xi_*^0)^2 + \xi_*^0\ln\left(\frac{\xi_0}{\xi_*^0}\right)\left[\frac{\xi_*^0}{2} - \frac{I_1(\xi_*^0)}{I_0(\xi_*^0)}\right]. \tag{8}$$

The time preceding the onset of boundary motion, i.e., the sublimation time of the coating along the initial boundary line ($\xi_* = \xi_*^0$), must be added to the above time intervals; as in [3], this time is determined by the formulas

$$\vartheta_{01} = 1 + (\xi_0 - \xi_*^0)\operatorname{th}\xi_*^0;\quad \vartheta_{11} = 1 + \xi_*^0\ln\left(\frac{\xi_0}{\xi_*^0}\right)\frac{I_1(\xi_*^0)}{I_0(\xi_*^0)}. \tag{9}$$

The total sublimation time $\vartheta_\nu^* = \vartheta_\nu + \vartheta_{\nu 1}$ is

$$\vartheta_0^* = 1 + \xi_0\xi_*^0 - \frac{1}{2}(\xi_*^0)^2;\ \vartheta_1^* = 1 + \frac{1}{2}\left[\ln\left(\frac{\xi_0}{\xi_*^0}\right) + \frac{1}{2}\right](\xi_*^0)^2. \tag{10}$$

The relation between the dimensional time t and the dimensionless time ϑ is given by the following expression in the present work, in contrast to [3]

$$\vartheta = \frac{\lambda_0}{\delta_0} \frac{T_* \delta\Theta_w}{\rho_0 \Delta_0 \Lambda} t.$$

If the internal surface of the gap is initially covered completely by the subliming layer, so that $\xi_0^* = \xi_0$, the sublimation time will depend quadratically on the length of the plane channel or the disk radius, since in this case

$$\vartheta_0^* = 1 + 0{,}5\xi_0^2, \quad \vartheta_1^* = 1 + 0{,}25\xi_0^2.$$

At very small ω (materials with a high latent heat of sublimation; for example, in water at the triple point, $\omega = 0.04457$) and moderate values of the dimensionless number $\omega_0 = 3(\mu L^2 T_* \lambda_0)/(h^3 p_* \delta_0)$ characterizing, in the familiar sense, the relation between the hydraulic resistance of the slot system and the thermal resistance of the walls, when $\xi_0^2 \ll 1$, the total sublimation time of the coating is practically determined by the first stage ($\vartheta_{\nu 1}$) in view of Eqs. (7)-(9), since the sublimation rate is almost the same over the whole extent of the slot system and is only limited by the thermal resistance of the walls; the hydraulic resistance of the sublimate flowing in the gap is negligibly small. As is evident from Eq. (9), $\vartheta_\nu^* = 1 + O(\xi_0^2)$ in this case.

NOTATION

$\lambda^{(\pm)}$, $\delta^{(\pm)}$, thermal conductivities and wall thicknesses of slot system; $T^{(\pm)}$, $J^{(\pm)}$, temperatures of external surfaces of these walls and flux densities of sublimate there; 2h, magnitude of gap; L, characteristic linear scale in median plane of slot system; μ, viscosity of sublimate at $T = T_*$; Λ, latent heat of sublimation; t, time of process; Δ_0, initial thickness of subliming coating; ρ_0, density of desublimate; p_*, T_*, characteristic pressure and temperature values of sublimate at saturation line; $x = L\xi$, $y = L\eta$, Cartesian coordinates in medium plane of gap; R, gas constant; σ_0, proportion of molecules reflected diffusely from walls; $\partial/\partial N$, derivative in the direction of the external normal to the contour bounding the perimeter of the slot system; $\nu = 0$ or 1 in the case of a plane channel or a gap between disks, respectively; $\lambda^{(\pm)} = \lambda_0$, $\delta^{(\pm)} = \delta_0$, $T^{(\pm)} = T_0$ with symmetric heat supply; $K^{(\pm)} = \lambda^{(\pm)}/\delta^{(\pm)}$; $\omega = RT_*/\Lambda$; $\omega_0 = (3/2)\cdot(\mu L^2 T_*/h^3 p_*)(K^{(+)} - K^{(-)})$; $\Pi = ph^2/\mu LV$; $\Pi_* = p_* h^2/\mu LV$; $\delta\Theta^{(\pm)} = \Theta^{(\pm)} - 1$; $\Theta = T/T_*$; $\delta\Theta_w = (K^{(+)}\delta\Theta^{(+)} + K^{(-)}\delta\Theta^{(-)})/(K^{(+)} + K^{(-)})$; $A_1 = -(\omega_0\omega^2)/(1 + \varepsilon)$; $A_0 = [(\omega_0\omega\Pi_*)/(1 + \varepsilon)]\delta\Theta_w$; V, characteristic velocity of sublimate motion; $\varepsilon = (3.78\cdot \mu\sqrt{RT_*}/hp_*)\cdot(2 - \sigma_0)/\sigma_0 - (9/4)\cdot(\mu^2 R^2 T^2_*/h^2 p^2_* \Lambda)$; $\xi_0^2 = -A_1$; $\Phi_\nu(\xi_*, \xi_*')$, function defined in Eq. (5); $\xi_* = \xi_0\xi$; $A_* = A_0/\xi_0^2 = (\Pi/\omega)\delta\Theta_w$; $J_m = (T_*/\Lambda)\cdot(\lambda_0/\delta_0)\delta\Theta_w$; $\xi'(t)$, coordinate of the boundary of the phase-transformation zone; $\vartheta = (\lambda_0/\delta_0)\cdot(T_*\delta\Theta/\rho_0\Delta_0\Lambda)t$, dimensionless time of process; $I_0(x)$, $I_1(x)$, Bessel function.

LITERATURE CITED

1. P. A. Novikov, L. Ya. Lyubin, and V. N. Denisov, Inzh.-Fiz. Zh., 54, No. 5, 806-813 (1988).
2. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Moscow (1953).
3. P. A. Novikov, L. Ya. Lyubin, and V. I. Novikova, Inzh.-Fiz. Zh., 52, No. 1, 73-80 (1987).